ÖRNEKLEME-I FİNAL CEVAP ANAHTARI

Cevap 1)

a)

$$p_{tbk} = \sum_{h=1}^{3} W_h \times p_h$$

$$p_1 = 0{,}6, \qquad p_2 = 0{,}4, \qquad p_3 = 0{,}5$$

$$W_1 = \frac{N_h}{N} = \frac{200}{400} = 0{,}5, \qquad W_2 = \frac{100}{400} = 0{,}25, \qquad W_3 = \frac{100}{400} = 0{,}25$$

$$p_{tbk} = 0{,}5 \times 0{,}6 + 0{,}25 \times 0{,}4 + 0{,}25 \times 0{,}5 = \mathbf{0,525}$$

$$\hat{V}(p_{tbk}) = \sum_{h=1}^{3} W_h^2 \hat{V}(p_h)$$

$$f_1 = \frac{10}{200} = 0{,}05, \qquad f_2 = \frac{10}{100} = 0{,}1, \qquad f_3 = \frac{10}{100} = 0{,}1$$

$$\hat{V}(p_1) = (1 - f_1)\frac{p_1(1-p_1)}{n_1} = (1 - 0{,}05)\frac{0{,}6 \times 0{,}4}{10} = 0{,}0228$$

$$\hat{V}(p_2) = (1 - 0{,}1)\frac{0{,}4 \times 0{,}6}{10} = 0{,}0216, \qquad \hat{V}(p_3) = (1 - 0{,}1)\frac{0{,}5 \times 0{,}5}{10} = 0{,}0225$$

$$\hat{V}(p_{tbk}) = (0{,}5)^2 \times 0{,}0228 + (0{,}25)^2 \times 0{,}0216 + (0{,}25)^2 \times 0{,}0225 = \mathbf{0,00845625}$$

b)

$$\hat{R} = \frac{\sum y_i}{\sum x_i} = \frac{\bar{y}}{\bar{x}}$$

$$\sum y_i = 26 + 27 + 28 = 81$$

$$\sum x_i = 48 + 72 + 70 = 190$$

$$\bar{x} = \frac{190}{30} = 6{,}333$$

$$\hat{R} = \frac{81}{190} = \mathbf{0,426315}$$

$$\hat{V}(\hat{R}) = \frac{(1-f)}{n \times \bar{x}}\frac{\sum\left(y_i - \hat{R}x_i\right)^2}{n}, f = \frac{n}{N} = \frac{30}{400} = 0{,}075$$

$$\hat{V}(\hat{R}) = \frac{(1-f)}{n \times \bar{x}^2}\frac{\sum y_i^2 - 2\hat{R}\sum x_i y_i + \hat{R}^2 \sum x_i^2}{n-1}$$

$$= \frac{(1-0{,}075)}{30 \times 6{,}333^2}\frac{73{,}6 + 76{,}5 + 81{,}2 - 2 \times 0{,}426315 \times (140 + 204 + 203) + (0{,}426315)^2 \times (294 + 552 + 548)}{29} = 0{,}0004841503$$

c)

$$n = \frac{N^2 t^2 S^2}{d^2 + N t^2 S^2}$$

$$S^2 = s^2(örneklem\ varyansı) = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} = 6{,}574713$$

$$90 = \frac{400^2 2^2 (6{,}574713)^2}{d^2 + 400 \times 2^2 (6{,}574713)^2}$$

$$d^2 = \frac{400^2 2^2 (6{,}574713)^2 - 90 \times 400 \times 2^2 (6{,}574713)^2}{90}$$

$$d^2 = 238228$$

$$d = \mathbf{488{,}086}$$

Cevap 2)

Örnekleme yöntemlerini duyarlılık yönünden karşılaştırabilmek için varyanslarına bakmak gerekir. En düşük varyansa sahip örnekleme yöntemi tercih edilmelidir.

$$f = 0$$

$$V_{brö}(\bar{y}) = (1-f)\frac{S^2}{n} = \frac{S^2}{n}$$

$$V_{orantılı}(\bar{y}) = \frac{(1-f)}{n}\sum_{h=1}^{L} W_h S_h^2 = \frac{1}{n}\sum_{h=1}^{L} W_h S_h^2$$

$$V_{neyman}(\bar{y}) = \frac{(1-f)}{n}\left(\sum_{h=1}^{L} W_h S_h\right)^2 = \frac{1}{n}\left(\sum_{h=1}^{L} W_h S_h\right)^2$$

$$S^2 = \frac{\sum_{h=1}^{L}(N_h - 1)S_h^2 + \sum_{h=1}^{L} N_h(\bar{Y}_h - \bar{Y})}{N-1}$$

$$\bar{Y} = \bar{Y}_{tb} = \sum_{h=1}^{L} W_h \bar{Y}_h$$

$$W_1 = \frac{5}{40} = 0{,}125, W_2 = \frac{15}{40} = 0{,}375, W_3 = \frac{20}{40} = 0{,}5$$

$$\bar{Y} = 0{,}125 \times 30 + 0{,}375 \times 50 + 0{,}5 \times 100 = 72{,}5$$

$$S^2 = \frac{4 \times 25^2 + 14 \times 20^2 + 19 \times 10^2 + 5 \times (30 - 72{,}5) + 15 \times (50 - 72{,}5) + 20 \times (100 - 72{,}5)}{39}$$

$$S^2 = 256{,}4103$$

$$V_{brö}(\bar{y}) = \frac{\mathbf{256{,}4103}}{\boldsymbol{n}}$$

$$V_{orantılı}(\bar{y}) = \frac{1}{n}(0{,}125 \times 25^2 + 0{,}375 \times 20^2 + 0{,}5 \times 10^2) = \frac{\mathbf{278{,}125}}{\boldsymbol{n}}$$

$$V_{neyman}(\bar{y}) = \frac{1}{n}(0{,}125 \times 25 + 0{,}375 \times 20 + 0{,}5 \times 10)^2 = \frac{\mathbf{244{,}1406}}{\boldsymbol{n}}$$

$V_{neyman}(\bar{y}) < V_{brö}(\bar{y}) < V_{orantılı}(\bar{y})$ olduğundan en uygun örnekleme yönteminin Neyman yöntemi olduğuna karar verilmiştir.

b)

En uygun yöntem Neyman yöntemi olarak bulunduğu için Neyman dağıtım yapılacaktır. $n = 20$ için tabakalara ait örneklem boyutları aşağıdaki gibi olur.

$$n_h = n\frac{N_h S_h}{\sum N_h S_h}$$

$$n_1 = 20\frac{5 \times 25}{5 \times 25 + 15 \times 20 + 20 \times 10} = \mathbf{4}$$

$$n_2 = 20\frac{15 \times 20}{5 \times 25 + 15 \times 20 + 20 \times 10} = 9{,}6 \cong \mathbf{10}$$

$$n_3 = 20\frac{20 \times 10}{5 \times 25 + 15 \times 20 + 20 \times 10} = 6{,}4 \cong \mathbf{6}$$

Cevap 3)

	Tabaka 1	Tabaka 2	Tabaka 3	Tabaka 4	
	8	11	7	15	
	11	8	11	12	
	12	9	15	11	
	8	3	13	10	
	10	13	12	12	
$\bar{y}_h$	$\frac{(8+11+\cdots+10)}{5} = 9{,}8$	8,8	11,6	12	$E(\bar{y}_{sis}) = \frac{(9{,}8+8{,}8+11{,}6+12)}{4} = 10{,}55$

$$\bar{Y} = \frac{\sum_{i=1}^{N} y_i}{N} = \frac{211}{20} = 10{,}55$$

$$\boldsymbol{E(\bar{y}_{sis}) = \bar{Y}}$$